Thermal Stress in a Layered Plate

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**Introduction**

This project tends to simulate the thermal stresses in a layered plate. The plate consists of a coating and a substrate. The coating is deposited onto the substrate at a temperature of 800 °C. The coating and the substrate are both stress-free at this temperature. The temperature is then lowered to 20 °C, and the thermal stress are examined. The material properties of the coating and the substrate are provided in Table 1:

|  |  |  |
| --- | --- | --- |
| **Property** | **Coating** | **Substrate** |
| Dimension [m] | 0.04\*0.04\*0.005 | 0.04\*0.04\*0.02 |
| Young’s modulus [GPa] | 70 | 130 |
| Poisson’s ratio | 0.17 | 0.28 |
| Density [kg/m3] | 1000 | 1000 |
| Thermal expansion coefficient [K-1] | 5e-7 | 3e-6 |

Table 1. Material properties.

**Theoretical Analysis**

The potential energy is:



from which the governing equation can be derived as:



where  is the body force,  is the surface traction,  is the Cauchy stress tensor,  is the elastic strain tensor:





where  is the linear elastic tensor,  is the linear strain tensor, and  is the thermal strain tensor:







**Finite Element Approximation**

Substituting Eq. , and Eq. into Eq. , the potential energy becomes:



图示

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Figure 1. Tetrahedron element (1st & 2nd order)

As shown in Figure 1, the first-order iso-parametric tetrahedron (4 nodes) is:



and the second-order iso-parametric tetrahedron (10 nodes) is:



Suppose each element has n nodes, then we may express the nodal displacement into a 3n\*1 vector:



and the displacement field inside an element is:



where  is the shape functions for node i as defined in Eq. and Eq. , and  is a 3\*3n matrix; therefore,  is 3\*1. The strain field is the derivative of the displacement field:



However, as  is a function of iso-geometric variables, Eq. cannot be directly applied. The relationship between the derivatives of global coordinates and iso-geometric coordinates is:





where  is the Jacobian matrix. Define a shape function 1\*n matrix :



then  is a 3\*m matrix:



The Jacobian matrix can be obtained from:



Now  can be calculated from , and it contains every non-zero element in  matrix. The potential energy of each element is:



where  is the thermal strain of each element which can be derived from Eq. following  defined in Eq. :



and  is the elastic constant matrix, which for a isotropic solid can be expressed as:



The minimum potential energy occurs at , yielding , where:





**Simulation Set Up**

As the plate is symmetric, only a quarter of it will be analyzed, as shown in Figure 2:

图示

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Figure 2. Model set up.

**Boundary Conditions**

In the case above, the body force can be neglected; assume the plate is traction free. Set  at  and  at . Set the displacement at the lower corner of substrate as zero, i.e., set  at .